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All the Four Dimensional Static, Spherically Symmetric Solutions of Abelian Kaluza-Klein Theory

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Abstract

We present the explicit form for all the four dimensional, static, spherically symmetric solutions in $(4+n)$-d Abelian Kaluza-Klein theory by performing a subset of $SO(2,n)$ transformations corresponding to four $SO(1,1)$ boosts on the Schwarzschild solution, supplemented by $SO(n)/SO(n-2)$ transformations. The solutions are parameterized by the mass M, Taub-Nut charge a, n electric $\vec{\mathcal{Q}}$ and n magnetic $\vec{\mathcal{P}}$ charges. Non-extreme black holes (with zero Taub-NUT charge) have either the Reissner-Nordström or Schwarzschild global space-time. Supersymmetric extreme black holes have a null or naked singularity, while non-supersymmetric extreme ones have a global space-time of extreme Reissner-Nordström black holes.

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Theories that attempt to unify gravity with other forces of nature in general involve, along with the graviton, additional scalar fields. Non-trivial 4-dimensional (4-d) configurations for such theories include a spatial variation of scalar fields, which in turn affects the space-time and thermal properties of such configurations. In particular, spherically symmetric solutions in Einstein-Maxwell-dilaton gravity have been studied extensively [1]. A subset of such configurations corresponds to black holes (BH's) which arise within effective (super)gravity theories describing superstring vacua. Configurations arising in the compactification of $(4+n)$-d gravity, *i.e.*, Kaluza-Klein (KK) theories [2], are also of interest since KK theory attempts to unify gravity with gauge interactions. In addition, such configurations can be viewed as a subset of BH's within the effective 4-d theory of heterotic superstring vacua [3,4].

In this letter, we find the explicit form for all the static, spherically symmetric solutions in $(4+n)$-d Abelian KK theory. These results as well as analogous results for BH's in effective string theory [5] were anticipated in Ref. [6], where the existence of a general class of solutions, which are obtained by appropriate generating techniques, was proven, however, without explicit calculations of the sort we shall present here. Such solutions can be generated by a subset of the $SO(2,n)$ ($\subset SL(2+n,\Re)$) transformations on the Schwarzschild solution. The explicit form of the 4-d space-time metric allows for the study of the global space-time and the thermal properties of such configurations. The study generalizes previous studies [7–9] of BH's in 5-d KK theory, as well as recent studies [10–12] of BH's with constrained charges in $(4+n)$-d Abelian KK theory. In addition, the work sets a stage for generating general axisymmetric solutions in KK theory [13] as well as in other sectors of supergravity theories [14].

The starting point is the effective 4-d Abelian KK theory obtained from $(4+n)$-d pure gravity by compactifying the extra n spatial coordinates on a torus by using the following KK metric Ansatz:

$$g^{(4+n)}_{\Lambda\Pi} \equiv \begin{bmatrix} \mathrm{e}^{-\frac{1}{\alpha}\varphi} g_{\mu\nu} + \mathrm{e}^{\frac{2\varphi}{n\alpha}} \rho_{ij} A^i_\mu A^j_\nu & \mathrm{e}^{\frac{2\varphi}{n\alpha}} \rho_{ij} A^j_\lambda \\ \mathrm{e}^{\frac{2\varphi}{n\alpha}} \rho_{ij} A^i_\pi & \mathrm{e}^{\frac{2\varphi}{n\alpha}} \rho_{ij} \end{bmatrix}, \tag{1}$$

where $g_{\mu\nu}$ is the 4-d Einstein frame metric [1] A^i_μ are n $U(1)$ gauge fields, ρ_{ij} is the unimodular part of the internal metric $g^{(4+n)}_{i+4,j+4}$ and $\alpha = [(n+2)/n]^{1/2}$.

Static or stationary solutions are invariant under the time-translation, which can be considered along with n internal $U(1)$ gauge transformations as a part of $(n+1)$-parameter Abelian isometry group generated by the commuting Killing vector fields $\xi^\Lambda_i := \delta^{i+3}_i$ ($i = 1, \cdots, n+1$) of a $(4+n)$-d space-time manifold M. In this case, the projection of the $(4+n)$-d manifold M onto the set S of the orbits of the isometry group in M allows one to express the $(4+n)$-d Einstein-Poincaré gravity action as the following effective 3-d one [15,7]:

$$\mathcal{L} = -\frac{1}{2}\sqrt{-h}[\mathcal{R}^{(h)} - \frac{1}{4}\mathrm{Tr}(\chi^{-1}\partial_a\chi\chi^{-1}\partial^a\chi)], \tag{2}$$

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[1]The convention for the signature of metric in this paper is $(+++-)$ with the time coordinate in the fourth component.

where $h_{ab} \equiv \tau g^{\perp}_{ab}$ $(a, b = 1, 2, 3)$ is the rescaled metric on S and

$$\chi \equiv \begin{bmatrix} \tau^{-1} & -\tau^{-1}\omega^T \\ -\tau^{-1}\omega & \breve{\lambda} + \tau^{-1}\omega\omega^T \end{bmatrix} \tag{3}$$

is the $(n+2) \times (n+2)$ symmetric, unimodular matrix of scalar fields on S. Here, $\breve{\lambda}_{ij} \equiv g^{(4+n)}_{\Lambda\Pi}\xi^{\Lambda}_i\xi^{\Pi}_j$, $\tau \equiv \det\breve{\lambda}_{ij}$ and $g^{\perp}_{ab} \equiv g^{(4+n)}_{ab} - \breve{\lambda}^{ij}\xi_{ia}\xi_{jb}$. The "potential" $\omega^T \equiv (\omega_1, ..., \omega_{n+1})$ defined as $\partial_a\omega_i = \omega_{ia} \equiv \hat{\epsilon}_{abc}\xi^{b;c}_i$ $(\hat{\epsilon}_{abc} \equiv \epsilon_{abc4...(4+n)})$ replaces the degrees of freedom of $\xi_{ia} = g^{(4+n)}{}_{i+3,a}$. The effective 3-d Lagrangian density (2) is invariant under the global $SL(2+n, \Re)$ target space transformations [15]:

$$\chi \to \mathcal{U}\chi\mathcal{U}^T, \qquad h_{ab} \to h_{ab}, \tag{4}$$

where $\mathcal{U} \in SL(2+n, \Re)$. In particular, the $SO(n)$ transformations [12] of the effective 4-d Lagrangian density constitute a subset of the $SL(2+n, \Re)$ transformations, which do not affect the 4-d space-time part of the metric.

The physically interesting solutions correspond to the configurations with an asymptotically $(|\vec{r}| \to \infty)$ flat 4-d space-time metric and constant values of the other 4-d fields. Without loss of generality one can take the Ansatz:

$$(g_{\mu\nu})_\infty = \eta_{\mu\nu},\ (A^i_\mu)_\infty = 0,\ \varphi_\infty = 0,\ (\rho_{ij})_\infty = \delta_{ij}, \tag{5}$$

which yields $\chi = \mathrm{diag}(-1, -1, 1, ..., 1)$.

The only subset of $SL(2+n, \Re)$ transformations (4), which preserves the asymptotic boundary conditions (5), is the $SO(2, n)$ transformation. A subset of $SO(2, n)$ transformations can then be used to act on known solutions to generate a new set of solutions of the equations of motion for the effective 3-d Lagrangian density (2).

In the following, we shall concentrate on static, spherically symmetric solutions. Spherical symmetry implies that for such configurations the metric h_{ab}, in polar coordinates (r, θ, ϕ), takes the form:

$$h_{ab} = \mathrm{diag}\left(1, f(r), f(r)\sin^2\theta\right), \tag{6}$$

where $a, b = r, \theta, \phi$, and χ depends only on the radial coordinate r. The transformation between the 3-d fields (h_{ab} and χ) and the corresponding 4-d fields is of the form:

$$\begin{aligned} \mathrm{e}^{-\frac{\varphi}{\alpha}} g_{\mu\nu} &= \mathrm{diag}(-\tau^{-1}, -\tau^{-1}f, -\tau^{-1}f\sin^2\theta, (\breve{\lambda}^{11})^{-1}), \\ \mathrm{e}^{\frac{2\varphi}{n\alpha}}\rho_{ij} &= \breve{\lambda}_{i+1,j+1}, \quad A^i_t = -\breve{\lambda}^{i+1,1}/\breve{\lambda}^{11}, \\ A^i_\phi &= \tau^{-1}f\cos\theta\,\mathrm{e}^{\frac{2\varphi}{n\alpha}}\rho^{ij}\partial_r\omega_{j+1}, \end{aligned} \tag{7}$$

with the constraint $\breve{\lambda}^{1k}\partial_r\omega_k = 0$ that the unphysical Taub-NUT charge is absent. Here, the spherically symmetric Ansatz for the 4-d metric is given by $g_{\mu\nu} = \mathrm{diag}(1/\lambda(r), R(r), R(r)\sin^2\theta, -\lambda(r))$, and the 4-d scalar fields φ and ρ_{ij} depend only on the radial coordinate r.

One way to generate the most general static, spherically symmetric solutions (with the Ansätze (7)) is by performing a subset of $SO(2, n)$ transformations on the 4-d Schwarzschild solution with the ADM mass m, which in terms of the 3-d quantities is of the following form:

$$\chi = \mathrm{diag}\left(-(1-\frac{m}{r})^{-1}, -(1-\frac{m}{r}), 1, ..., 1\right), \tag{8}$$

and $f(r) = r(r-m)$. The subset of $SO(2,n)$ transformations that generates new types of solutions is the quotient space $SO(2,n)/SO(n)$ [2]. The $2n+1$ parameters of $SO(2,n)/SO(n)$ along with the parameter m constitute the $2n+2$ parameters, which correspond to the mass M, n electric $\vec{\mathcal{Q}}$ and n magnetic $\vec{\mathcal{P}}$ charges as well as the Taub-Nut charge a of the most general, spherically symmetric, stationary solution in $(4+n)$-d KK theory. In fact, each representative of the elements of $SO(2,n)/SO(n)$ generates a physical parameter of the solution [3]: n boosts on the first [or the second] index of χ (of the Schwarzschild solution) and on one of the last n indices of χ generate magnetic [or electric] charges, and an $SO(2)$ rotation on the first two indices of χ generates an unphysical Taub-NUT charge a.

For the purpose of obtaining the explicit form of static, spherically symmetric solutions with a general charge configuration, it is convenient to first perform two successive $SO(1,1)$ boosts on the $1st$ and $(n+1)$-th, and the $2nd$ and $(n+2)$-th indices of (8) with the boost parameters $\delta_{P,Q}$, respectively, yielding:

$$\chi = \begin{bmatrix} -\frac{r+\hat{P}}{r} & 0 & \cdot & \frac{|P|}{r} & 0 \\ 0 & -\frac{r+2\beta-\hat{Q}}{r+2\beta} & \cdot & 0 & \frac{|Q|}{r+2\beta} \\ \cdot & \cdot & \mathbf{I} & \cdot & \cdot \\ \frac{|P|}{r} & 0 & \cdot & \frac{r+2\beta-\hat{P}}{r} & 0 \\ 0 & \frac{|Q|}{r+2\beta} & \cdot & 0 & \frac{r+\hat{Q}}{r+2\beta} \end{bmatrix}, \tag{9}$$

and $f(r) = r(r-2\beta)$. Here, $\beta \equiv m/2$ and $\hat{Q} = \beta + \sqrt{Q^2+\beta^2}$ [$\hat{P} = \beta + \sqrt{P^2+\beta^2}$], where $P \equiv m\mathrm{sinh}\delta_P\mathrm{cosh}\delta_P$ [$Q \equiv m\mathrm{sinh}\delta_Q\mathrm{cosh}\delta_Q$]. $\mathbf{I}$ is the $(n-2)\times(n-2)$ identity matrix, $\cdot$ denotes the zero entries, and the event horizon r_+ is shifted to the origin ($r=0$). The solution (9) corresponds to the $U(1)_M \times U(1)_E$ BH solutions [4] with the ADM mass $M = \hat{P}+\hat{Q}$, the physical magnetic [electric] charge P [Q], and $\beta \geq 0$ measuring a deviation from the supersymmetric limit [10].

A class of new solutions can be obtained by performing $SO(n)/SO(n-2)$ transformations, parameterized by $2n-3$ parameters, on (9). Such transformations act on the lower-right $n\times n$ part of χ and, thus, do not affect the 4-d space-time metric $g_{\mu\nu}$ and the dilaton φ. The transformed solutions have n electric $\vec{\mathcal{Q}}$ and n magnetic $\vec{\mathcal{P}}$ charges, subject to one constraint $\vec{\mathcal{P}}\cdot\vec{\mathcal{Q}} = 0$.

Thus, in order to generate the most general, static, spherically symmetric solution one needs only one more parameter, associated with $SO(2,n)/SO(n)$ transformations. Such a

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[2] All the axisymmetric stationary solutions can be generated by performing $SO(2,n)/SO(n)$ transformations on the Kerr solution [13].

[3] Similar observations are due to Gibbons [16].

[4] These solutions were first found in Refs. [11,12] by directly solving the equations of motion with a diagonal internal metric Ansatz.

parameter is provided by two $SO(1,1)$ boosts on the $1st$ and $(n+2)$-th, and the $2nd$ and $(n+1)$-th indices of χ in (9), whose respective boost parameters δ_1 and δ_2 have to be related to one another in order to yield solutions with no Taub-NUT charge. The transformed solutions are of the form:

$$
\begin{aligned}
\lambda &= \frac{r(r+2\beta)}{(XY)^{1/2}}, \quad R = (XY)^{1/2}, \quad e^{\frac{2\varphi}{\alpha}} = \frac{X}{Y}, \\
\rho_{ij} &= \delta_{ij}\mathrm{e}^{-\frac{2\varphi}{n\alpha}}, \quad \rho_{n-1,n-1} = \frac{We^{\frac{2(n-2)}{n\alpha}\varphi}}{(XY)^{1/2}}, \\
\rho_{n-1,n} &= \frac{Ze^{\frac{2(n-2)}{n\alpha}\varphi}}{(XY)^{1/2}}, \ \rho_{n,n} = \frac{(r+\hat{Q})(r+\hat{P})}{(XY)^{1/2}}e^{\frac{2(n-2)}{n\alpha}\varphi},
\end{aligned}
\tag{10}
$$

where

$$
\begin{aligned}
X &= r^2 + [(2\beta - \hat{P} + \hat{Q})\cosh^2\delta_2 + \hat{P}]r + 2\beta\hat{Q}\cosh^2\delta_2, \\
Y &= r^2 + [(2\beta + \hat{P} - \hat{Q})\cosh^2\delta_1 + \hat{Q}]r + 2\beta\hat{P}\cosh^2\delta_1, \\
W &= r^2 + [(2\beta + \hat{P} - \hat{Q})\cosh^2\delta_1 + (2\beta - \hat{P} + \hat{Q})\cosh^2\delta_2]r \\
&\quad + 2[\beta(2\beta - \hat{P} - \hat{Q}) + \hat{P}\hat{Q}]\cosh^2\delta_1\cosh^2\delta_2 \\
&\quad + (2\beta - \hat{Q})\hat{P}\cosh^2\delta_1 + (2\beta - \hat{P})\hat{Q}\cosh^2\delta_2 \\
&\quad + |P||Q|\cosh\delta_1\cosh\delta_2\sinh\delta_1\sinh\delta_2, \\
Z &= [|P|\sinh\delta_1\cosh\delta_2 + |Q|\sinh\delta_2\cosh\delta_1]r \\
&\quad + |P|\hat{Q}\sinh\delta_1 + \hat{P}|Q|\sinh\delta_2,
\end{aligned}
\tag{11}
$$

with the non-zero electric and magnetic charges and the ADM mass given by

$$
\begin{aligned}
P_{n-1} &= |P|\cosh\delta_1\cosh\delta_2 + |Q|\sinh\delta_1\sinh\delta_2, \\
P_n &= -(\hat{P} - \hat{Q} + 2\beta)\cosh\delta_1\sinh\delta_1, \\
Q_{n-1} &= -(\hat{P} - \hat{Q} - 2\beta)\cosh\delta_2\sinh\delta_2, \\
Q_n &= |Q|\cosh\delta_1\cosh\delta_2 + |P|\sinh\delta_1\sinh\delta_2, \\
M &= (2\beta + \hat{P} - \hat{Q})\cosh^2\delta_1 + (2\beta + \hat{Q} - \hat{P})\cosh^2\delta_2 \\
&\quad + \hat{P} + \hat{Q} - 4\beta.
\end{aligned}
\tag{12}
$$

Here, the electric fields are given by $E_i = R^{-1}\mathrm{e}^{-\alpha\varphi}\rho^{ij}Q_j$ $(i = 1, ..., n)$. The requirement $\breve{\lambda}^{1k}\partial_r\omega_k = 0$, *i.e.*, the unphysical Taub-NUT charge a is zero, relates the two boost parameters $\delta_{1,2}$ in the following way:

$$
|P|\tanh\delta_2 + |Q|\tanh\delta_1 = 0. \tag{13}
$$

Thereby, the transformed solutions (10) are parameterized by 4 independent parameters, *i.e.*, the non-extremality parameter [5] β, the electric Q and magnetic P charges of the $U(1)_M \times$

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[5]When the non-extremality parameter β is zero and the other parameters are kept finite, the no-Taub-NUT-charge condition (13) ensures $\vec{\mathcal{P}} \cdot \vec{\mathcal{Q}} = 0$, *i.e.*, this is a condition satisfied by supersymmetric configurations [10].

$U(1)_E$ solution, and the boost parameters $\delta_{1,2}$, subject to the constraint (13). The resultant solution is in turn specified by the mass [6] M and four charges, however only three of them are independent.

The remaining $2n-3$ degrees of freedom, required to parameterize the most general, static, spherically symmetric BH's in Abelian $(4+n)$-d KK theory, are then provided by $SO(n)/SO(n-2)$ rotations on the solutions (10).

We shall now analyze the global space-time structure and the thermal properties of the above solution. Since $SO(n)/SO(n-2)$ rotations on (10) do not change the 4-d space-time (as well as φ and the scalar product $\vec{\mathcal{P}} \cdot \vec{\mathcal{Q}}$), it is sufficient to consider the solutions (10) for the purpose of determining the space-time and thermal properties for all the $(4+n)$-d Abelian KK BH's. Without loss of generality, we assume that $|Q| \geq |P|$. In the case of $|Q| \leq |P|$, the roles of (δ_1, δ_2) and (P, Q) are interchanged.

We first discuss the singularity structure. Non-extreme solutions ($\beta > 0$) always have a space-time singularity behind or at $r = -2\beta$. Namely, the space-time singularity, *i.e.*, the point at which $R(r) = 0$, where the Ricci scalar $\mathcal{R}$ blows up, occurs at the real roots of $X(r)$ and $Y(r)$, which are always $\leq -2\beta$ with equality holding when $P = 0$ or $\delta_2 = 0$. On the other hand, $\lambda(r)$ is zero at $r = 0$ and $r = -2\beta$, provided $X(r)$ and $Y(r)$ do not have roots at these points, *i.e.*, when $\delta_2 \neq 0$ and $P \neq 0$, in which case $r = 0$ and $r = -2\beta$ correspond to the outer and inner horizons, respectively.

The extreme limit ($\beta \to 0$) with the other parameters finite corresponds to supersymmetric BH's with the singularity at $r = 0$. The singularity is null, *i.e.*, $r = 0$ is also the horizon, except when $P = 0$, in which case the singularity becomes naked. The extreme limit ($\beta \to 0$) with $|Q| \to |P|$, while keeping $\beta \mathrm{e}^{2|\delta_2|} \equiv 2|q|$ and $||Q| - |P||\mathrm{e}^{2|\delta_2|} \equiv 4|\Delta|$ finite, corresponds to nonsupersymmetric BH's with the global space-time of extreme Reissner-Nordström BH's.

Thermal properties of solutions (10) are specified by the 4-d space-time at the outer horizon located at $r = 0$. The Hawking temperature [17] $T_H = |\partial_r \lambda(r = 0)|/4\pi$ is given by

$$T_H = \frac{1}{4\pi \left(\hat{P}\hat{Q}\right)^{1/2} \cosh\delta_1 \cosh\delta_2} = \frac{[|Q|^2 \cosh^2\delta_2 - |P|^2 \sinh^2\delta_2]^{1/2}}{4\pi \left(\hat{P}\hat{Q}\right)^{1/2} |Q| \cosh^2\delta_2}. \tag{14}$$

As the boost parameter δ_2 increases the temperature T_H decreases, approaching zero temperature. In the supersymmetric extreme limit and with zero P, the temperature is always infinite independently of δ_2. In the non-supersymmetric extreme limit, the temperature is zero.

The entropy [18] S of the system, determined as $S = \frac{1}{4} \times$(the surface area of the event horizon), is of the following form:

$$S = 2\pi\beta \left(\hat{P}\hat{Q}\right)^{1/2} \cosh\delta_1 \cosh\delta_2$$

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[6]When no-Taub-NUT-charge condition (13) is imposed, the mass M is compatible with the corresponding Bogomol'nyi bound: $M \geq |\vec{\mathcal{P}}| + |\vec{\mathcal{Q}}|$.

$$= \frac{2\pi\beta\left(\hat{P}\hat{Q}\right)^{1/2}|Q|\cosh^2\delta_2}{[|Q|^2\cosh^2\delta_2 - |P|^2\sinh^2\delta_2]^{1/2}}. \tag{15}$$

The entropy increases with δ_2, approaching infinity [finite value] as $\delta_2 \to \infty$ [non-supersymmetric extreme limit is reached]. In the supersymmetric extreme limit, the entropy is zero.

We now summarize the results according to the values of parameters δ_2, P and β:

- Non-extreme BH's with $\delta_2 \neq 0$, $P \neq 0$ [7]: The global space-time is that of non-extreme Reissner-Nordström BH's, *i.e.*, the time-like singularity is hidden behind the inner horizon. The temperature T_H [entropy S] is finite, and decreases [increases] as δ_2 or β increases, approaching zero temperature [infinite entropy].

- Non-extreme BH's with $\delta_2 = 0$ or $P = 0$: The singularity structure is that of the Schwarzschild BH's, *i.e.*, the space-like singularity is hidden behind the (outer) horizon. The tempterature T_H [entropy S] is finite and decreases [increases] as β increases, approaching zero [infinity].

- Supersymmetric extreme BH's, *i.e.*, δ_2 finite: For $P \neq 0$, the solution has a null singularity, which becomes naked when $P = 0$. The temperature T_H [entropy S] is finite and becomes infinite [zero] when $P = 0$.

- Non-supersymmetric extreme BH's , *i.e.*, $|\delta_2| \to \infty$ with (q,Δ) non-zero:latex The global space-time is that of extreme Reissner-Nordström BH's with zero temperature T_H and finite entropy S [8].

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[7]Note, that non-extreme BH's of 5-d KK theory belong to this class. They are obtained from solutions (10) by performing an $SO(2)$ rotation on the $(n+1)$-*th* and $(n+2)$-*th* indices of the corresponding matrix χ, however, the corresponding rotation parameter is related to δ_2.

[8]Extreme dyonic solutions of 5-d KK theory [9] are obtained from this one by choosing an $SO(2)$ rotation angle, related to Q, q and Δ.

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